

Find the following limits.

Each answer should be a number, ∞ , $-\infty$, or DNE (only if the other answers do not apply).

SCORE: ____ / 7 PTS

$\frac{\infty}{-\infty}$ INDETERMINATE

[a] $\lim_{x \rightarrow 1^+} \frac{4-x^2}{1-x^4} = -\infty$ (1)

$\frac{3}{1-1^+} \rightarrow \frac{3}{0^-}$ NO POINTS IF YOUR ONLY ANSWER IS "DNE"

[b] $\lim_{x \rightarrow -\infty} \frac{\sqrt{7x^6-5}}{13x^3+11x^2}$

$$= \lim_{x \rightarrow -\infty} \frac{\sqrt{7x^6-5}}{13x^3+11x^2} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}}$$

$$= \lim_{x \rightarrow -\infty} \frac{\sqrt{7x^6-5}}{13x^3+11x^2} \cdot \frac{-\sqrt{\frac{1}{x^6}}}{\frac{1}{x^3}}$$
 (1)

$$= \lim_{x \rightarrow -\infty} \left[\frac{-\sqrt{7-\frac{5}{x^6}}}{13+\frac{11}{x}} \right]$$
 (1)

[c] $\lim_{x \rightarrow \infty} (e^{-x} - 6 \tan^{-1} x)$

$$= \lim_{x \rightarrow \infty} e^{-x} - 6 \lim_{x \rightarrow \infty} \tan^{-1} x$$

$$= 0 - 6 \cdot \frac{\pi}{2}$$
 (1)

$$= -3\pi$$
 (1)

$$= \frac{-\sqrt{7-0}}{13+0}$$

$$= \frac{-\sqrt{7}}{13}$$
 (1)

The graph of $f(x)$ is shown on the right.

SCORE: _____ / 6 PTS

[a] Find all x -coordinates where $f(x)$ is not differentiable, and explain very briefly why.

① $x = -3$ DISCONTINUITY

① $x = 2$ CUSP

① $x = 4$ VERTICAL TANGENT LINE

[b] Sketch a graph of $f'(x)$ on the same axes.



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Determine if the function below is continuous at $x = -1$.

SCORE: _____ / 4 PTS

State your conclusion clearly, and show whether each condition in the definition of "continuous (at a point)" is true or false.

In addition, if it is not continuous, determine the type of discontinuity, and state the conditions in the definition of that type of discontinuity that are true.

$$f(x) = \begin{cases} x^3 - x^4 - x^6, & \text{if } x < -1 \\ -3, & \text{if } x = -1 \\ x^7 + x^5 - x, & \text{if } x > -1 \end{cases}$$

$\frac{1}{2}$
f HAS A JUMP DISCONTINUITY @ $x = -1$
SINCE $\lim_{x \rightarrow -1^+} f(x)$ AND $\lim_{x \rightarrow -1^-} f(x)$ BOTH EXIST
 \uparrow BUT ARE NOT EQUAL

$\frac{1}{2}$ $f(-1) = -3$ EXISTS.

$\lim_{x \rightarrow -1^+} f(x) = \lim_{x \rightarrow -1^+} (x^7 + x^5 - x) = -1 - 1 + 1 = -1$ $\frac{1}{2}$

$\lim_{x \rightarrow -1^-} f(x) = \lim_{x \rightarrow -1^-} (x^3 - x^4 - x^6) = -1 - 1 - 1 = -3$ $\frac{1}{2}$

$\lim_{x \rightarrow -1} f(x)$ DNE $\frac{1}{2}$

SO, f IS NOT CONTINUOUS
AT $x = -1$ $\frac{1}{2}$

Let $f(x) = \frac{x}{8-x}$.

SCORE: _____ / 8 PTS

[a] Find $f'(x)$.

$$\begin{aligned} & \textcircled{1} \left| \lim_{h \rightarrow 0} \frac{\frac{x+h}{8-(x+h)} - \frac{x}{8-x}}{h} \right| \\ &= \lim_{h \rightarrow 0} \frac{(x+h)(8-x) - x(8-x-h)}{h(8-x-h)(8-x)} \\ &= \left| \lim_{h \rightarrow 0} \frac{8x + 8h - x^2 - hx - 8x + x^2 + xh}{h(8-x-h)(8-x)} \right| \textcircled{\frac{1}{2}} \\ &= \left| \lim_{h \rightarrow 0} \frac{8}{(8-x-h)(8-x)} \right| = \left| \frac{8}{(8-x)^2} \right| \textcircled{1} \end{aligned}$$

[b] Find the slope-point form of the equation of the tangent line to the curve of $f(x)$ at the point where $x = 6$.

$$\text{SLOPE} = f'(6) = \frac{8}{2^2} = 2 \textcircled{1}$$

$$\text{POINT} = (6, f(6)) = (6, \frac{6}{2}) = (6, 3)$$

$$\underline{y - 3 = 2(x - 6)} \textcircled{1}$$

[c] The position of an object moving along a line is $s(t) = \frac{t}{8-t}$ inches, where t is the time in seconds.

Find the instantaneous velocity of the object at time $t = 2$ seconds. Give the correct units for your answer.

$$s'(2) = \frac{8}{6^2} = \left| \frac{2}{9} \right| \text{ INCHES/SECOND} \textcircled{\frac{1}{2}}$$

$\textcircled{1}$

Your score on a test depends on how much time you spent studying for it the day before the test.

SCORE: _____ / 2 PTS

Suppose $P = f(s)$, where P is your score on the test, in points, and s is the number of hours you studied for it the day before.

What does $f'(3) = 5$ mean? Your answer must use all the numbers from that equation, and the correct units for those numbers.

NOTE: Your answer must NOT use "slope", "change" nor "derivative".

IF YOU STUDIED 3 HOURS THE DAY BEFORE THE TEST,

YOU WOULD SCORE 5 MORE POINTS FOR EACH ADDITIONAL HOUR

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YOU HAD STUDIED

Using complete sentences & proper mathematical notation,

SCORE: _____ / 3 PT

write the formal definitions of the following terms as shown in lecture.

[a] derivative (function)

THE DERIVATIVE OF f IS $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

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[b] continuous (at a point)

f IS CONTINUOUS AT a IF $f(a)$ EXISTS, $\lim_{x \rightarrow a} f(x)$ EXISTS

AND $\lim_{x \rightarrow a} f(x) = f(a)$